Birmingham City Schools

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Algebra II with Trigonometry
4^{th} Nine Weeks Curriculum Guide
Week 6

Suggested Pacing: 1 50 min class

ACOS Standards:
37 – Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle
38 – Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles transversed counterclockwise around the unit circle
39 – Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions

Learning Objectives:
Students will find the coordinates of points on the unit circle.

Key Vocabulary:
Unit circle - a circle of radius 1 whose center is at the origin of the coordinate plane
Quadrantal angle – an angle that has its terminal side lying along an x or y-axis
Cosine of Θ – the x-coordinate of the point at which the terminal side of the angle intersects the unit circle
Sine of Θ - – the y-coordinate of the point at which the terminal side of the angle intersects the unit circle
Tangent Θ – the ratio $\frac{y}{x}$ when the terminal side of an angle Θ in standard position intersects the unit circle
cosecant Θ – the ratio $\frac{1}{y}$ when the terminal side of an angle Θ in standard position intersects the unit circle
secant Θ – the ratio $\frac{1}{x}$ when the terminal side of an angle Θ in standard position intersects the unit circle
cotangent Θ – the ratio $\frac{x}{y}$ when the terminal side of an angle Θ in standard position intersects the unit circle

Assessment Plan:

Formative: Teacher will walk around and monitor student work on guided practice. Teacher will assess student understanding of the concept as the independent practice is checked.

Learning Activities:

Before:
Review past vocabulary and define quadrantal angles.

*There is a good applet at www.mathopenref.com that demonstrates the different types of angles formed on a unit circle. Click on angles in trigonometry and choose the type of angle you want to model.

During:
The teacher will start off by demonstrating how to build the table below to help students learn the values of the sine and cosine for the quadrantal angles and multiples of 30° $\left(\frac{\pi}{6}\right)$, 45° $\left(\frac{\pi}{4}\right)$, and 60° $\left(\frac{\pi}{3}\right)$. Note: you get the same values from the $30°,60°,90°$ and $45°,45°,90°$ special right triangles.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sine	0	1	2	3	4
cosine	4	3	2	1	0

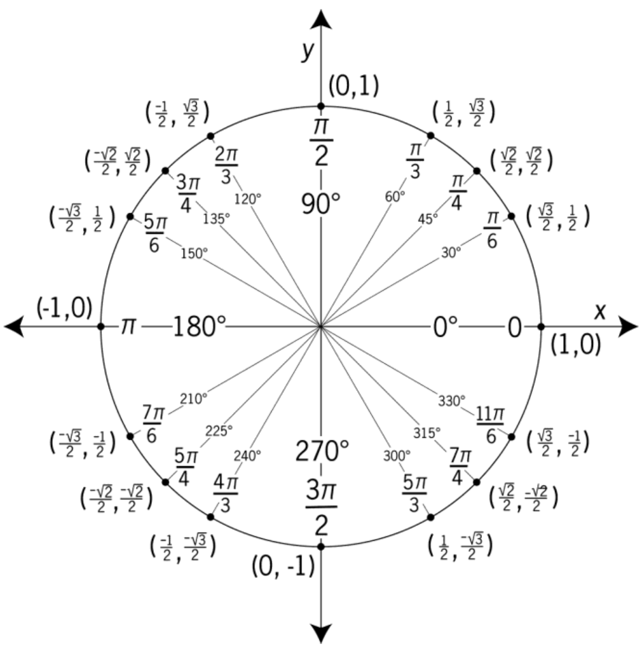
Start with the table above. Divide every number (0-4) by 2. Take the square root of the top. Simplify. This gives you the table below.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Next, the teacher will define the cosine and sine of Θ and then model how to fill in the ordered pairs in the first quadrant of the unit circle using the table above and all quadrantal angles.

Then, discuss the following:

1. The sign of x and y in each quadrant.
   Q1: (+, +), Q2: (-,+), Q3: (-,-), Q4: (+,-)
2. How the values for the sine and cosine of the angles in the first quadrant are related to the multiples of these angles in the other quadrants (same values – just different signs).



3. Define the secant, cosecant, and tangent of ϴ.

Work the examples below with the students.

Example 1: (#1)

$$\tan 60° = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

Example 2: (#2)

$$\sin 225° = -\frac{\sqrt{2}}{2}$$

Example 3: (#15)

$$\csc 225^\circ = \frac{1}{-\frac{\sqrt{2}}{2}} = 1 \cdot \frac{-2}{\sqrt{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

After:
Have the students do a guided practice: WS – "Exact Trig Ratios of Important Angles" #3-18
Assign students an independent practice: complete the worksheet

Materials:
paper, pencil, unit circle, worksheet "Exact Trig Ratios of Important Angles"

Differentiation/Accommodations:
Cut down on the number of problems that you have the students work or do not make them completely simplify the answers.

Technology Integration:
Calculators (preferably one that has a fraction button)

Teacher Notes:
You may need to review how to rationalize the denominator.

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39 – Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions

Learning Objectives:
Graph the sine curve.
Graph the cosine curve.
Graph the tangent curve.

Key Vocabulary:
Sine function – matches the measure Θ of an angle in standard position with the y-coordinate of a point on the unit circle
Sine curve – the graph of the sine function
Cosine function - matches the measure Θ of an angle in standard position with the x-coordinate of a point on the unit circle
Tangent function - matches the measure Θ of an angle in standard position with the y/x ratio of the (x, y) coordinates of a point on the unit circle

Assessment Plan:
Formative: Teacher will walk around and monitor student work on guided practice. Teacher will assess student understanding of the concept as the independent practice is checked.

Learning Activities:

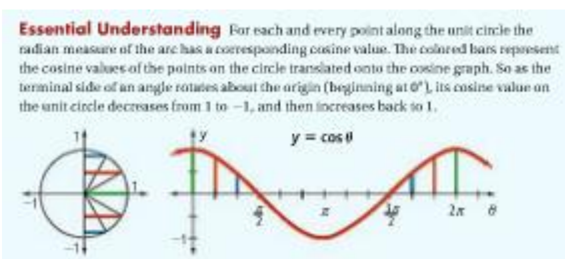
Before:

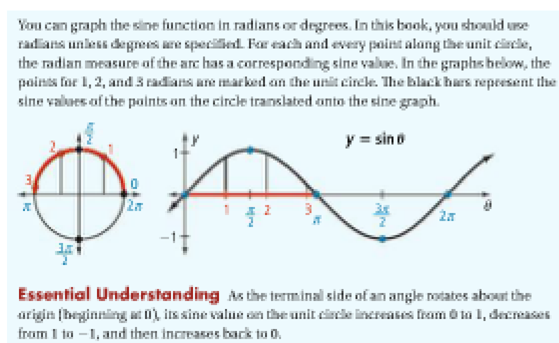
Introductory Activity – Unwrapping the Unit Circle

1. Divide the students into groups of 2-4 for this activity.
2. Assign each group to unwrap either the cosine or the sine graph.

3. Give each group a sheet of graph paper and some wikki sticks. (See the teacher notes for how to label the units on the x- and y-axis of the graph paper.)
4. To unwrap the unit circle to get the cosine graph, use the wikki sticks to measure the horizontal distance from the y-axis to the point where the terminal side of each angle intersects the unit circle. Cut the sticks to the length of the horizontal distance.
   To unwrap the unit circle to get the sine graph, use the wikki sticks to measure the vertical distance from the x-axis to the point where the terminal side of each angle intersects the unit circle. Cut the sticks to the length of the vertical distance.
5. Repeat the steps above for all of the quadrantal angles and angles that are multiples of $\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3}$ from $[0,2\pi]$.
6. Place the wikki sticks above (if the sine or cosine is positive for that angle) or below (if the sine or cosine is negative for that angle)their corresponding angles.
7. The curve formed by placing the horizontal and vertical distances above and below the x-axis shows the graph of the sine and cosine functions. (See snippet below)

The activity is described in the textbook on pages 851 and 861. The snippets below are from the textbook.





3. Give each group a sheet of graph paper and some wikki sticks. (See the teacher notes for how to label the units on the x- and y-axis of the graph paper.)
4. To unwrap the unit circle to get the cosine graph, use the wikki sticks to measure the horizontal distance from the y-axis to the point where the terminal side of each angle intersects the unit circle. Cut the sticks to the length of the horizontal distance.
   To unwrap the unit circle to get the sine graph, use the wikki sticks to measure the vertical distance from the x-axis to the point where the terminal side of each angle intersects the unit circle. Cut the sticks to the length of the vertical distance.
5. Repeat the steps above for all of the quadrantal angles and angles that are multiples of

$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ from $[0, 2\pi]$.

6. Place the wikki sticks above (if the sine or cosine is positive for that angle) or below (if

Next, the teacher will demonstrate how to find the values for tangent, cosecant, secant, and cotangent using the definitions of these trig functions from the key vocabulary from yesterday's lesson.

After:

The students will continue to complete the tables for cosine, tangent, secant, cosecant, and cotangent. Have the students to focus on the getting the values for $[0, 2\pi]$ done and plotted so that the teacher can assist as needed.

For homework, the students should complete the tables and graphs for all 6 trig functions.

Materials:

Unit circle, worksheets for the graphs of the 6 trig functions, graph paper, wikki sticks or bendables (these will stick to the graph paper)
Note: uncooked spaghetti may be used in place of the wikki sticks. You will have to tape it down if you want it to stick.

Differentiation/Accommodations:

If needed, you can have the students complete the graphs from only $[0, 2\pi]$ instead of $[-2\pi, 2\pi]$.

Technology Integration:

Calculators

Teacher Notes:

The unit circle is divided into sectors each measuring 15^0 or $\left(\frac{\pi}{12}\right)$ radians. Each unit on the x-axis of the graph paper for the 6 trig functions also represents 15^0 or $\left(\frac{\pi}{12}\right)$ radians. Let 4 units on the y-axis be 1. Have the students to label the x-axis with all the quadrantal angles and angles that are multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ from $[-2\pi, 2\pi]$.

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Week 6

Suggested Pacing: 1 day 90 min class (Example)

ACOS Standards:
38 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

40 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Learning Objectives:
Students will identify properties of the sine, cosine, and tangent function.
Students will graph sine, cosine, and tangent curves.

Key Vocabulary:
sine function- $y = \sin\theta$, matches the measure θ of an angle in standard position with a y –coordinate of a point on the unit circle.
sine curve- graph of a sine function.
amplitude- half the difference of the maximum and minimum values of a periodic function.
period- one complete cycle.
five point pattern- zero, max, zero, min, zero.
cosine function- $y = \cos\theta$ matches θ with the x-coordinate of the point on the unit circle where the terminal side of angle θ intersects the unit circle.
zero- where the graph of a function crosses the x-axis.
tangent function- is the ratio of sine over cosine.
tangent of θ- the ratio of $\frac{y}{x}$.

Assessment Plan: Teacher will monitor students and take up guided practice for a grade.

Learning Activities:

Before:
Teacher will graph the sine function on the board from 0 to 2π. The teacher will post the following questions:
What point is the maximum?
What point is the minimum?
What are the zeros?

Answers:
Maximum: $\left(\frac{\pi}{2},\ 1\right)$
Minimum: $\left(\frac{3\pi}{2},\ -1\right)$
Zeros: $(0,\ 0)$; $(\pi,\ 0)$; $(2\pi,\ 0)$

Teacher will graph the cosine function on the board from 0 to 2π. The teacher will post the following questions:
What point is the maximum?
What point is the minimum?
What are the zeros?

Answers:
Maximum: $(0,\ 1)$; $(2\pi,\ 1)$
Minimum: $(\pi,\ -1)$
Zeros: $\left(\frac{\pi}{2},\ 0\right)$; $\left(\frac{3\pi}{2},\ 0\right)$

Tell students that these are the five point patterns for the sine and cosine functions. We will use these numbers to stretch and shrink the graphs of the functions.

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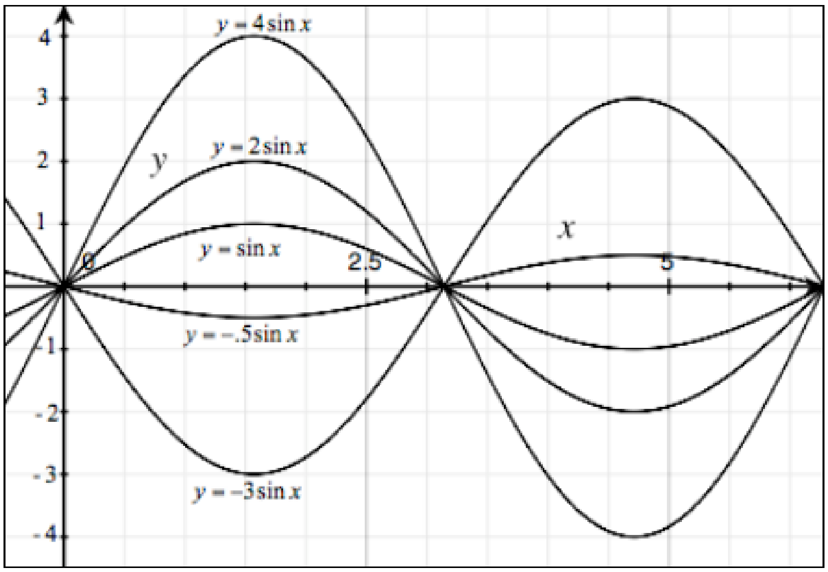
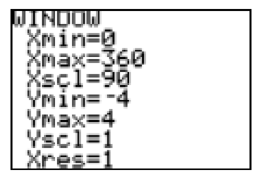
During:

Graph this for students on Gizmo:

On your calculators, graph the curves $y = \sin x, y = 2\sin x, y = 4\sin x, y = -.5\sin x, y = -3\sin x$ in degree mode on the window below.





Notice that by changing the coefficient of the function, we control its scaling factor – a vertical stretch or vertical shrink of the basic sine curve. We call this the amplitude of the curve – the height of the curve above its axis of symmetry.

The **amplitude** of $y = a\sin x$ or $y = a\cos x$ is the largest value of y and is given by $|a|$.

The range of the curve is $[-a, a]$.

We define the shape of the curve using this chart: $\begin{cases} a > 0 \ldots \text{curve normal} \\ a < 0 \ldots \text{curve reversed} \end{cases}$

We define vertical change using this chart: $\begin{cases} \text{amplitude } > 1 \ldots \text{vertically stretch} \\ \text{amplitude } < 1 \ldots \text{vertically shrunk} \\ \text{amplitude } = 1 \ldots \text{no vertical change} \end{cases}$

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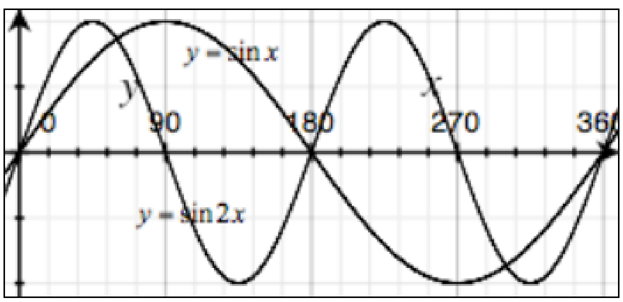
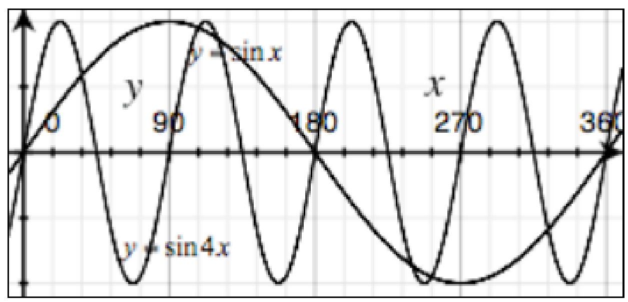
Example 1) For each of the curves below, find the amplitude, range, and 5 critical points (using degrees). Then graph it on the calculator using an appropriate window to show the behavior of the graph.

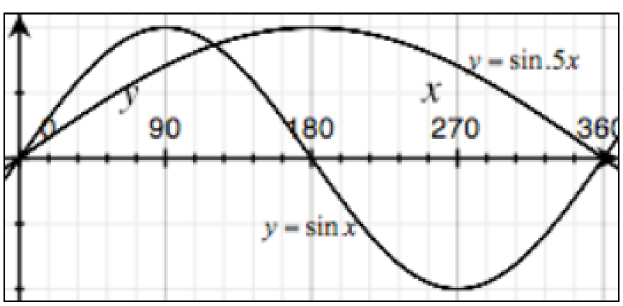
	Curve	Amplitude	Range	Critical Points	Shape (circle)	Vertical (circle)
a.	$y = 3\cos x$				Normal / Reverse	Stretch / Shrink
b.	$y = -2\sin x$				Normal / Reverse	Stretch / Shrink
c.	$y = \cos x$				Normal / Reverse	Stretch / Shrink
d.	$y = 7\sin x$				Normal / Reverse	Stretch / Shrink
e.	$y = \frac{2}{3}\sin x$				Normal / Reverse	Stretch / Shrink
f.	$y = -\pi\cos x$				Normal / Reverse	Stretch / Shrink

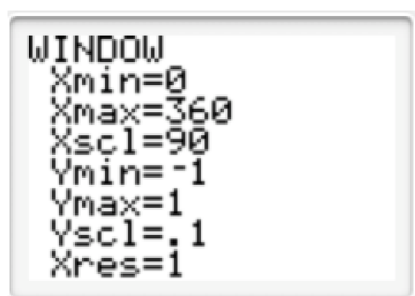
Graph this for students on Gizmo:

On your calculators, graph the curves $y = \sin x, y = \sin 2x, y = \sin 4x, y = \sin .5x$ in degree mode on the window on the next page.









Notice that by changing the coefficient of the x-variable, we control the number of cycles of the curve we produce. In $y = \sin x$, when the coefficient of x is 1, we get one cycle within 360°. In $y = \sin 2x$, we get two cycles within 360°. Thus we also change the **period** of the curve, the number of degrees (or radians) the curve takes to complete one cycle.

The **period** of $y = \sin bx$ or $y = \cos bx$ is the number of degrees (or radians) the curve takes to complete one cycle. It is found using this formula: $\text{Period} = \begin{cases} \dfrac{360^\circ}{b} & \text{for degrees} \\ \dfrac{2\pi}{b} & \text{for radians} \end{cases}$

We define the horizontal stretch in words: $\begin{cases} \text{period } < 360^\circ \ (\text{or } 2\pi) \ \ldots \text{ compressed} \\ \text{period } > 360^\circ \ (\text{or } 2\pi) \ \ldots \text{ elongated} \\ \text{period } = 360^\circ \ (\text{or } 2\pi) \ \ldots \text{ none} \end{cases}$

To find the 5 critical points, we use $0, \dfrac{\text{period}}{4}, \dfrac{\text{period}}{2}, \dfrac{3\text{period}}{4}, \text{period}$ whether we are in degrees or radians.

Example 2) For each of the curves below, find the period (use degrees), horizontal stretch, and 5 critical points. Then graph it on the calculator using an appropriate window to show the behavior of the graph.

	Curve	**Period**	**Horizontal Stretch (circle)**	**Critical Points**
a.	$y = \cos 2x$		Compressed / elongated	
b.	$y = \sin 3x$		Compressed / elongated	
c.	$y = \cos 4x$		Compressed / elongated	
d.	$y = \sin 0.5x$		Compressed / elongated	
e.	$y = \cos \frac{2}{3}x$		Compressed / elongated	
f.	$y = \sin \frac{3}{2}x$		Compressed / elongated	
g.	$y = \cos 10x$		Compressed / elongated	

After:

Example 3) Now lets put both amplitude and period together. For each of the curves below, find all the pertinent information.

	Curve	Amplitude	Period	Critical Points	Shape (circle)	Vertical Stretch (circle)	Horizontal Stretch (circle)
a.	$y = 2\sin x$				Normal Reversed	Stretched Shrunk	Compressed Elongated
b.	$y = \sin 2x$				Normal Reversed	Stretched Shrunk	Compressed Elongated
c.	$y = 2\sin 2x$				Normal Reversed	Stretched Shrunk	Compressed Elongated
d.	$y = 4\cos 3x$				Normal Reversed	Stretched Shrunk	Compressed Elongated
e.	$y = -3\cos 6x$				Normal Reversed	Stretched Shrunk	Compressed Elongated
f.	$y = \frac{1}{2}\sin\frac{1}{2}x$				Normal Reversed	Stretched Shrunk	Compressed Elongated
g.	$y = \sin\frac{4}{3}x$				Normal Reversed	Stretched Shrunk	Compressed Elongated
h.	$y = -\frac{3}{4}\sin\frac{3}{4}x$				Normal Reversed	Stretched Shrunk	Compressed Elongated

Materials:
Pencil, paper, copies of the charts

Differentiation/Accommodations:
Allow students to work together to complete Example 3.

Technology Integration:

Use the Gizmo for trigonometric functions.

Teacher Notes:
Students have to learn to stretch and shrink sine and cosine functions. It will take practice.

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Week 6

Suggested Pacing: 1 day 50 min class

ACOS Standards:
40 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Learning Objectives:
Students will write the equations of sine and cosine functions.

Key Vocabulary:
Same as the lesson before.

Assessment Plan:
Teacher will monitor students as they take notes and do their guided practice. The practice can be taken for a grade.

Learning Activities:

Before:

The teacher will write the following equations on the board:

$f(x) = -2\sin 4x$

$g(x) = \frac{1}{4}\cos 3x$

$h(x) = 5\tan\frac{1}{2}x$

Ask the students for the amplitude and period of each function.

Answers:

The amplitude of f(x) is 2. The period of f(x) is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The amplitude of g(x) is $\frac{1}{4}$. The period of g(x) is $\frac{2\pi}{3}$.

The amplitude of h(x) is 5. The period of h(x) is $\frac{\pi}{\frac{1}{2}} = 2\pi$.

During:

We will now have the students write the equations of the three trig functions, when given the amplitude and period.

Example 1

Write the equation of a sine curve given that the amplitude is 2 and the period is $\frac{\pi}{3}$.

Use the basic equation $y = a\sin b\theta$, where a is the amplitude and b is the period.

The answer would be $y = 2\sin\frac{\pi}{3}\theta$.

Example 2

Write the equation of a cosine curve given that the shape is reversed with an amplitude of $\frac{1}{4}$ and a period of 5π.

Use the basic equation $y = a\cos b\theta$, where a is the amplitude and b is the period.

The answer would be $y = -\frac{1}{4}\cos 5\pi\theta$

After:

Have students do p. 856 (15-20)

Materials:
paper, pencil, textbook

Differentiation/Accommodations:
Scaffold with stronger students helping the weaker students.

Technology Integration:
None

Teacher Notes:
The students are working backwards from the classwork on the previous day.